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Damped Oscillations and Time Constant

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Background

Non-damped Simple Harmonic Motion

Oscillation is, in physics, a periodic fluctuation between two measurements. Typically, the simplest way of observing an oscillation is by attaching a mass to the end of a spring, where the mass is subject to a force of gravity.

The dynamics of the spring-mass system can be well described by the laws of *Simple Harmonic Motion*, which is ideally defined as the motion of a simple harmonic oscillator whose motion is not damped. The obtained graph for the position along the time is a sinusoid, as can be observed in figure 1.

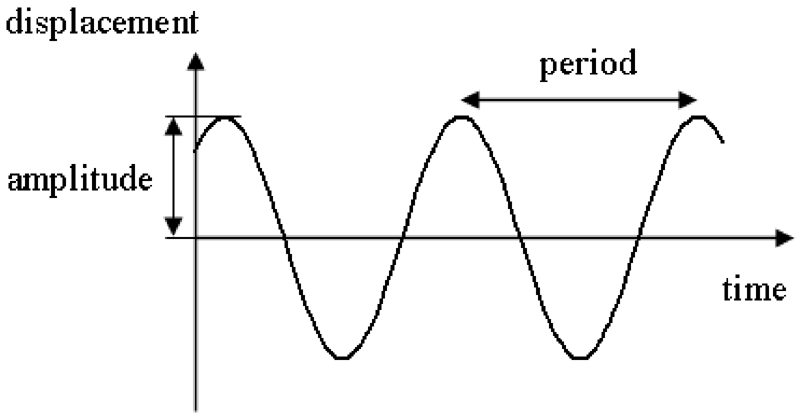

Figure 1: Simple Harmonic Motion, non-damped oscillation.

Admittedly, in Simple Harmonic Motion, the motion is periodic, since it repeats the same cycle at regular intervals of time with a similar behavior, and hence being a sinusoid, as mentioned above. Therefore, and as it can be observed in figure 1, Simple Harmonic Motion is a displacement along the time, with an amplitued (highest and lowest points) and a period (time of repetition for an entire cycle). The sinusoidal waves are given by the expression:

$$y(t) = A\sin(\omega t + \phi) = A\sin(2\pi f t + \phi) \tag{1}$$

In formula 1, $y(t)$ is the displacement along the time, where A is the amplitude of the sinusoid, f is the frequency of the wave, t is the elapsed time and ϕ is the phase of the oscillation. Furthermore the period of a sinusoid is given by equation 2:

$$T = \frac{1}{f} \tag{2}$$

Damped Oscillations

All the background given in the previous section is the very base of theoretical approaches and calculations on basic Simple Harmonic Motion calculations. However, the systems referred about are based on ideal oscillations, which have no damping at all.

Non-damped oscillation means that the sinusoid will repeat it periodicity forever, meaning that the oscillator itself is obviously not affected by external forces and hence it remains in a continuous movement loop. Nevertheless, in real experiments[1] the spring often comes to rest in a relatively short period of time.

This fact is easily accepted when the drag force[2] of the air is taken in account. Astonishingly, damped oscillations decrease with a theoretically measurable ratio, which can be described mathematically introducing a concept denominated *time constant*, denoted by the symbol τ. Consider figure 2 in order to observe the actual behavior of a spring oscillation in normal conditions (damped oscillation).

[1] This team performed another set of experiments last year about non-damped oscillations and still discovered that the spring would not oscillate forever, but instead it became to stop at the rest point within some finite amount of time.

[2] For small velocities, the resisting drag force F_d is given by: $F_d = -\gamma v$.

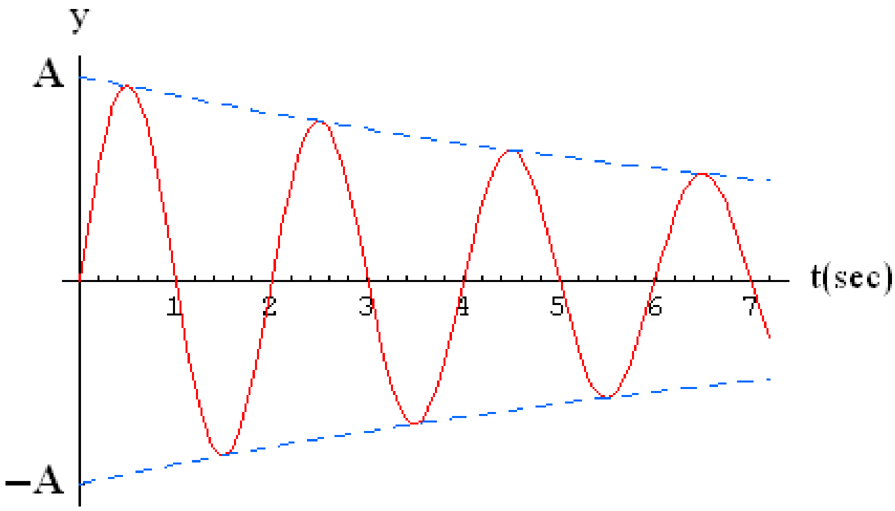

Figure 2: Simple Harmonic Motion, damped oscillation.

According to figure 2, the damped oscillation behaves as stated before, however it can be observed the green line which is a fitted curve describing the dampness ratio of the wave, which is enforced by an exponential function dependent of the time constant τ. This curve is given by equation 3.

$$A(t) = A_0 e^{\frac{-t}{\tau}} \tag{3}$$

Thus, the amplitude $A(t)$ decreases exponentially in time as a function of τ, where the time constant is related to the coefficient $\gamma = \frac{2m}{\tau}$. Further, by letting $t = \tau$ in equation 3, it can be seen that the amplitude has a value of $A(\tau) = A_0 e^{-1} \approx 0.37 A_0$. This fact means that the amplitude has decreased to the 37% of its initial value (when $t = 0$).

Purpose and Description of the Experiment

The purpose of this experiment is to analyze the damped oscillation of a spring with an attached mass and then try to manually find an exponential curve to the oscillation graph so that it fits the decreasing ratio of dampness. Moreover, the team is encouraged to check how the practical results vary from theoretical calculations. The ultimate goal is to see, if there are discrepancies between theory and practice, which could be the main reasons.

For this experiment, the team used a spring and an attached mass to it, a relatively long stand where to hang the spring, a motion sensor available in the laboratory and the measurement software Logger Pro 345.

First of all, the team set the equipment up, placing the spring hanging from the stand and putting the motion sensor beneath the mass attached to the spring. The minimal distance for proper function is set to 40cm at least. In addition, before going into the direct analysis, the team performed a series of tests in order to check that the obtained result from the oscillation was a smooth wave. This initial phase was carried out by recording waves for a few seconds and adjusting the motion sensor so that the wave became smoother.

Moreover, the team initially measured the distance from the position at rest, so that it could later be subtracted from the displacement measurements. The position at rest for this very experiment was at 46 centimeters. As the minimal distance for proper function of the motion sensor is 40 centimeters, the team performed only mild oscillations by just pulling the mass a little bit, thus resulting in a soft and smooth wave with low amplitude, which always helps to avoid the effect of clipping.

Once the wave was smooth enough (and hence the clipping was avoided) the team set the Logger Pro software to automatically record up to $6minutes$ after pressing the *Start* button. After pressing *Start*, the team waited for six minutes until the end of the data recording. Once the oscillation was recorded as well as it dampness, the team proceeded to further analyze the obtained data and perform the calculations.

Experiment Performance

When the team gathered the damped oscillation during six minutes, the graph clearly told that the spring was become to rest by showing a very low amplitude at the end of the graph, according to figure 3.

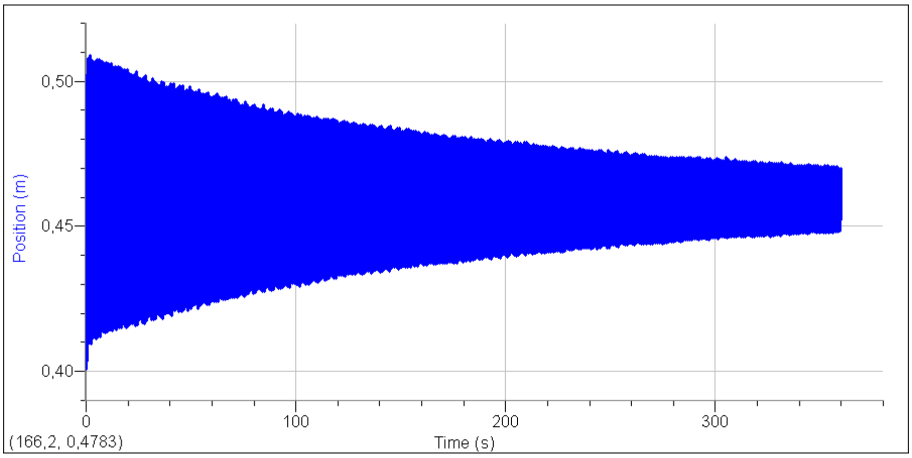

Figure 3: Damped oscillation measured. Position respect to the motion sensor.

Furthermore, from figure 4 can be depicted the real displacement from the rest point (or equilibrium point) of the spring, which was calculated by adding a new data column into the data table of Logger Pro, giving the expression for the displacement as $displacement = position - 0.46m$, since the initial distance from the equilibrium point to the motion sensor must be taken in account. The real displacement curve obtained is shown in figure 4.

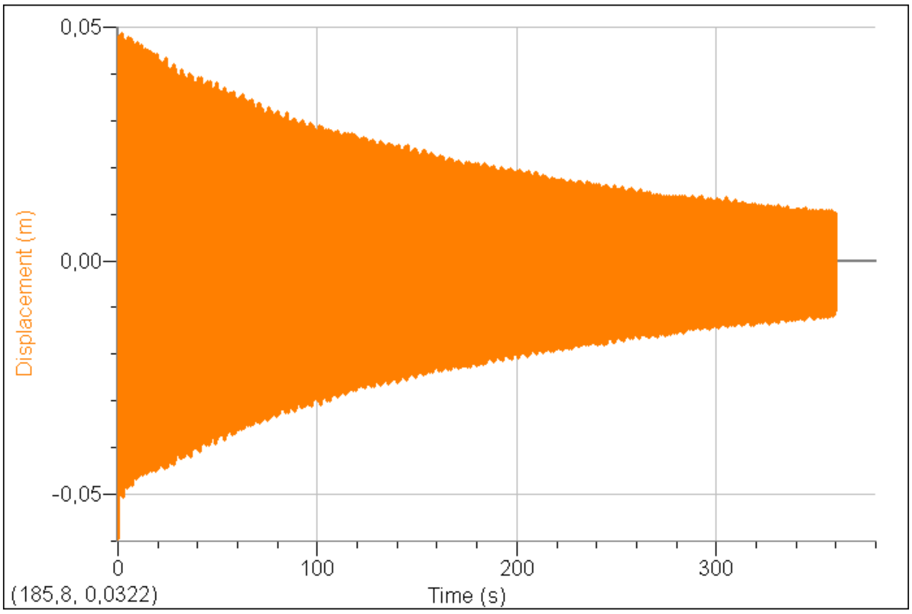

Figure 4: Actual amplitude of the damped oscillation respect to the equilibrium point.

Depicting from figure 4, it can be safely said that the amplitude of this very performance is 5cm from the equilibrium point. As stated in the background, above in this paper, it is known that $\tau = 0.37A$, and hence it can be estimated the time constant from figure 4, when the amplitude is exactly a 37% of its initial value. The initial amplitude is admittedly 5cm, and because τ is found in figure 4 when the amplitude is $0.37 \cdot 5cm = 1.85cm$, then the team positioned the cursor at the amplitude peak of 1.85cm and depicted the value for the time (from the x-axis), which turned out to be $\tau = 213s$.

Results

Consequently, the team proceeded to add a new calculated data column with the formula of the amplitude in function of the time constant, given in equation 3. This column incorporated a whole list of new values which are actually a list of points for the fitting curve of the damped oscillation. After adding the column, the team added the corresponding graph with the curve fit. The fitting curve and the displacement graphs can be compared in figure 5.

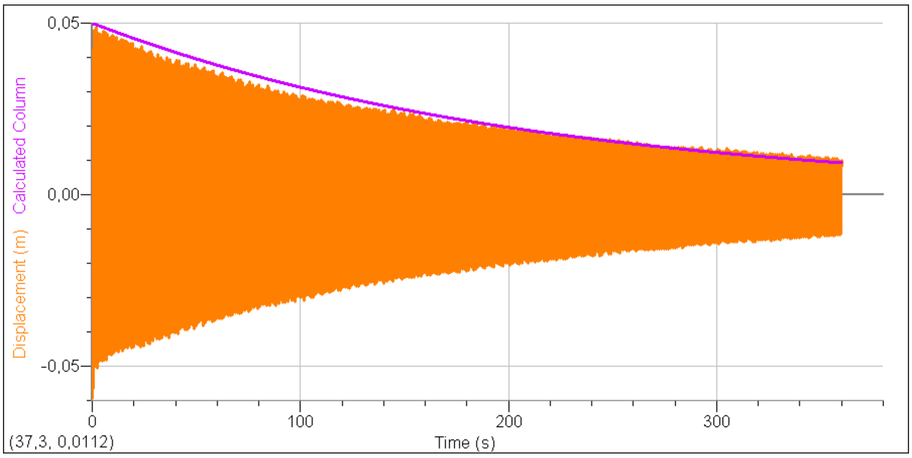

Figure 5: Damped oscillation and its fitting exponential curve.

Accordingly, the fitting curve is quite precise, however there are a couple of points to be taken in account. First, at the beginning of the graph, it can be observed that there is a tiny gap between the curve and the amplitude, meaning that the precision here is not accurate at all times. Furthermore, it can be seen from the end of the graph that the curve buries itself into the amplitude sketch, being slightly lower than the amplitude, which again confirms that the curve fit is not exactly accurate at some points of the damped oscillation.

This observations question if the practical results are what they were expected to be or not. After several minutes of analysis and discussion among the members of the team, there was an answer to the question. Surprisingly, the form of the damped oscillation does not exactly fit on its theoretical dampness curve fit due to the nature of the drag force exerted by the air in the environment. This is due to the fact that the drag force, in this case, is not proportional to the velocity of the oscillator, and hence the drag force is considered to be a result of a so-called *turbulence*.

Conclusion

In general, the experiment was quite easy to carry out. As usual, the most tedious part was to set the equipment and configure it in a manner so that the team could collect proper information on the Logger Pro display. Once the equipments where configured and the team gathered a smooth curve, the rest was pure fun.

One point to comment out is that the team first performed a manual fit by using a different approach than the one stated in the experiment instructions. Actually, the team managed to set up a manual τ and adjust it several times until obtaining a very fitting curve. However, then the team proceeded to add a new calculated data column with the actual formula. However, it was surprising that the team could manually "invent" an own τ for which the curve was quite exact.

Further analysis about the irregularities in figure 5 were discussed also with the supervisor and we were told why the drag force behaved in such a way, so that the fitting curve could almost never correspond with the dampness of the oscillation, due to the *turbulent* nature.

From the learning view point, it is quite rewarding to perform this kind of experiment, because our Applied Physics course provides us with theory about oscillators and dampness, but it is enriching for engineering students to actually perform experiments and contrast the results with the theory discussed in the lectures. Consequently, letting students to become motivated and have a general understanding of what is going on. As usual, me and my team are looking forward to "put our hands on" the following experiments.